Proposition: **The H index emerges within an oligopoly model as an endogenously determined, by the degree of collusion the number of firms and the distribution of costs, variable.**

Proof:

Clarke & Davies (1982) model:

From last lecture the profit maximizing condition was written

$$P\left(1 - \frac{(1+\lambda_i)}{\eta}S_i\right) = MC_i \tag{1}$$

Set

$$\frac{dQ_j}{Q_j} = \alpha \frac{dQ_i}{Q_i} \text{ for all } j\neq i \text{ and for all } i$$

$0\leq \alpha \leq 1$. As α (the degree of implicit collusion) tends to 1 we tend to perfect collusion (joint profit maximization) and as it tends to 0 we tend to the Cournot case. So,

$$\frac{dQ_j}{dQ_i} = \alpha \frac{Q_j}{Q_i} \Rightarrow \frac{d\sum_{j\neq i}Q_j}{dQ_i} = \alpha \frac{\sum_{j\neq i}Q_j}{Q_i} \Rightarrow \lambda_i = \alpha\left(\frac{Q}{Q_i} - 1\right) \Leftrightarrow$$

$$1 + \lambda_i = \alpha(1/S_i) - \alpha + 1 \tag{2}$$

Substitutiting (2) into (1)

$$P\left(1 - \frac{1}{\eta}(S_i - \alpha S_i + \alpha)\right) = MC_i \Leftrightarrow$$

$$S_i = -\frac{\alpha}{1-\alpha} + \frac{\eta}{1-\alpha}\left(1 - \frac{MC_i}{P}\right) \tag{3}$$

Summing over the N firms in the industry and solving for price gives

$$P = \left(\sum MC_i\right) \eta [N(\eta-\alpha)-(1-\alpha)]^{-1}$$

Substituting this into (3), which is then squared and summed gives:

$$H = (1/N) + \left(1 - N\frac{(\eta-\alpha)}{(1-\alpha)}\right)^2 \frac{CV_c^2}{N}$$

where CV_c is the coefficient of variation of marginal costs across firms. So the level of concentration depends on the degree of collusion within the industry (conduct), the number of firms and the distribution of costs. The intuition behind this result is that for a given set of demand and cost curves, the largest firms tend to benefit more from the collusive restriction of output, and thereby size inequalities are further increased (provided N is constant).