1 PATENT RACING -THE GAME THEORETIC APPROACH

Assume two firms A and B who want to decide whether they should attempt to produce a new product with marginal cost c. The demand for the good is $P = a - bQ$ and as for the R&D effort, the costs of setting up a research lab are K and the probability that the lab will successfully develop the product is ρ. If both firms successfully develop the product they will be a Cournot duopoly ($\pi_A = \pi_B = \frac{(a-c)^2}{9b} = \frac{4M}{9}$, $P^C = \frac{a+2c}{3}$, $q_1^C = q_2^C = \frac{a-c}{3b}$), while if only one of the two develops the new product she will be rewarded with monopoly profits ($M = \frac{(a-c)^2}{4b}$, $P^M = \frac{a+c}{2}$, $Q^M = \frac{a-c}{2b}$).

The expected profit net of setup costs if only A establishes an R&D lab while B does not is

$$\begin{aligned} E(\pi_B \quad &| \quad R\&D_A = 0, R\&D_B > 0) = E(\pi_A \mid R\&D_A > 0, R\&D_B = 0) = \\ &= \rho\frac{(a-c)^2}{4b} + (1-\rho)0 - K = \rho M - K \end{aligned}$$

If both establish R&D labs, then

$$\begin{aligned} E(\pi_B \quad &| \quad R\&D_A > 0, R\&D_B > 0) = E(\pi_A \mid R\&D_A > 0, R\&D_B > 0) = \\ &= \rho(1-\rho)\frac{(a-c)^2}{4b} + \rho^2\frac{(a-c)^2}{9b} - K = \rho M(1 - \frac{5}{9}\rho) - K \end{aligned}$$

	No R&D	R&D
No R&D	$0, 0$	$0, \rho M - K$
R&D	$\rho M - K, 0$	$\rho M(1 - \frac{5}{9}\rho) - K, \rho M(1 - \frac{5}{9}\rho) - K$

1.1 Possible Nash equilibrium outcomes

1. Neither firm establishes an R&D lab if $\rho M < K$.

2. Only one firm establishes an R&D division in equilibrium, say A if

a. (R&D, no R&D) > (no R&D, no R&D) (if $\rho M > K$) and

b. (R&D, no R&D) > (R&D, R&D)

(if $0 > \rho M(1 - \frac{5}{9}\rho) - K \iff K > \rho M(1 - \frac{5}{9}\rho)$.

Combining a and b gives:

$$\rho M > K > \rho M(1 - \tfrac{5}{9}\rho)$$

3. Both firms will R&D in Nash equilibrium if $\rho M(1 - \frac{5}{9}\rho) > K$. In this case B prefers to R&D irrespective of whether A R&Ds or not and the same applies for A. Hence the (R&D, R&D) is a dominant strategies equilibrium.

1.2 Complications

Note that (R&D,R&D) may be a Nash equilibrium,but it will not necessarily be a Pareto optimum from the firms' perspective if the sum of profits under this equilibrium is less than the sum of profits under a (R&D, no R&D) or (no R&D, R&D) combination. This will be the case if

$$2\rho M(1-\frac{5}{9}\rho)-2K<\rho M-K \Longleftrightarrow K>\rho M(1-\frac{10}{9}\rho)$$

and combining this with the fact that (R&D,R&D) is a Nash equilibrium, gives:

$$\rho M(1-\frac{5}{9}\rho)>K>\rho M(1-\frac{10}{9}\rho)$$

Hence the above condition gives us the case where both firms doing R&D is a Nash equilibrium which is not Pareto optimal from the firms' point of view, in the sense that firms would be jointly better off with only one of the two firms having an R&D lab; in other words there is "too much"[1] innovation.

1.3 Consumer surplus

In the case of monopoly consumer surplus is equal to

$$CS_M=\frac{(a-\frac{a+c}{2})(\frac{a-c}{2b})}{2}=\frac{M}{2}$$

while in the Cournot case it is equal to

$$CS_C=2\frac{(a-\frac{a+2c}{3})(\frac{a-c}{3b})}{2}=2\frac{4M}{9}=\frac{8M}{9}$$

The expected social surplus ignoring the R&D costs if only one firm, say A, established an R&D lab is equal to:

$$E(\pi_A \mid R\&D_A>0,R\&D_B=0)+E(\pi_B \mid R\&D_A>0,R\&D_B=0)+\rho CS_M$$
$$=\rho M+0+\rho\frac{M}{2}=\frac{3}{2}\rho M$$

The expected social surplus ignoring the R&D costs if both firms set up an R&D lab are

$$E(\pi_A \mid R\&D_A>0,R\&D_B>0)+E(\pi_B \mid R\&D_A>0,R\&D_B>0)$$
$$+\rho^2 CS_C+2\rho(1-\rho)S_M=$$

[1] In other words, "too much" is defined as the case where both firms innovate as it is a Nash equilibrium to do so, but this generates less aggregate profit than if only firm only performed R&D.

$$= 2[\rho(1-\rho)M + \rho^2\frac{4M}{9}] + \rho^2\frac{8M}{9} + 2\rho(1-\rho)\frac{M}{2} = \rho M(3 - \frac{11\rho}{9})$$

The second lab is socially desirable only if

$$\rho M(3 - \frac{11\rho}{9}) - K - K > \frac{3}{2}\rho M - K \Longleftrightarrow K < \rho M(1.5 - \frac{11\rho}{9})$$

So a Nash equilibrium of both firms doing R&D (where $K < \rho M(1 - \frac{5\rho}{9})$ will be Pareto optimum from the society's point of view if

$$K < \rho M(1.5 - \frac{11\rho}{9})$$

for $\rho > 0.75$ (since then $\rho M(1.5 - \frac{11\rho}{9}) < \rho M(1 - \frac{5\rho}{9})$), and always for $\rho < 0.75$. On the other hand, both firms doing R&D will not be socially optimum (would be "too much") from the society's point of view if

$$\rho M(1 - \frac{5\rho}{9}) > K > \rho M(1.5 - \frac{11\rho}{9})$$

for $\rho > 0.75$.

Corollary 1 *It is always socially optimum for both firms to do R&D if the probability of success is low.*

Corollary 2 *If both firms doing R&D is desirable (Pareto optimal) from the firms' point of view because* $K < \rho M(1 - \frac{10}{9}\rho)$, *then since* $\rho M(1.5 - \frac{11}{9}\rho) > \rho M(1 - \frac{10}{9}\rho)$, *it always follows that double R&D will also be optimum from the society's point of view.*

Corollary 3 *It is possible to have too much innovation from the firms' point of view, while it is socially optimum from the society's point of view if*

$$\rho M(1.5 - \frac{11}{9}\rho) > K > \rho M(1 - \frac{10}{9}\rho)$$